1. Evaluate the limit:
$\lim_{x\to 0} \dfrac{\ln(1+4x)}{2x}$
   A. 0
   B. 1
   C. 2
   D. ∞
   E. None of the above

2. If $x^3 - xy + y^3 = 1$ and y is a dif–ferentiable function of x, find $\dfrac{dy}{dx}$.
   A. $\dfrac{3x^2}{x-3y^2}$
   B. $\dfrac{3x^2-1}{1-3y^2}$
   C. $\dfrac{y}{3y^2-x}$
   D. $\dfrac{3x^2-y}{x-3y^2}$
   E. None of the above

3. Evaluate $\int xe^x\,dx$.
   A. $e^x + C$
   B. $xe^x + C$
   C. $xe^x - e^x + C$
   D. $\frac{1}{2}x^2e^x + C$
   E. None of the above

4. Solve the first-order differential equation
$\frac{dy}{dx} = \frac{y}{x} + 1$ for $x > 0$.

A. $y = Cx + x \ln x$

B. $y = Cx + x$

C. $y = Cx + (\ln x)^2$

D. $y = Cx + \ln x$

E. None of the above

5. A sequence $\{a_n\}_{n=1}^{\infty}$ satisfies

$$a_n = \begin{cases} 1, & \text{if } n = 1, 4, 7, \ldots, 3k+1, \ldots \\ -1, & \text{if } n = 2, 5, 8, \ldots, 3k+2, \ldots \\ 0, & \text{otherwise} \end{cases}$$

For $\delta \in [0, 1)$, find $\lim_{m \to \infty} \sum_{n=1}^{m} \delta^n a_n$.

A. $\frac{\delta - \delta^2}{1 + \delta^3}$

B. $\frac{2\delta - \delta^2}{1 + \delta + \delta^2}$

C. $\frac{1 + \delta}{1 + \delta + \delta^3}$

D. $\frac{\delta}{1 + \delta + \delta^2}$

E. None of the above

6. Let $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & -2 & 1 \\ 1 & 2 & 1 \end{bmatrix}$. Find $\det(A^{-1})$.

A. 7

B. -7

C. $\frac{1}{7}$

D. $-\frac{1}{7}$

E. None of the above

7. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$. Is $\mathbf{b} = \begin{bmatrix} 2 \\ 3 \\ 2 \\ 3 \end{bmatrix}$ in $\text{span}\{\mathbf{v}_1, \mathbf{v}_2\}$?

A. Yes, and $\mathbf{b} = 2\mathbf{v}_1 + 3\mathbf{v}_2$

B. Yes, and $\mathbf{b} = 3\mathbf{v}_1 + 2\mathbf{v}_2$

C. Yes, and $\mathbf{b} = 2\mathbf{v}_1 + 2\mathbf{v}_2$

D. No, $\mathbf{b}$ is not in the span

E. None of the above

8. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$.

Compute $A\mathbf{x} + \mathbf{b}$.

A. $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

B. $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$

C. $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

D. $\begin{bmatrix} -1 \\ -1 \end{bmatrix}$

E. None of the above

9. Let $A = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{bmatrix}$. Which of the following is an eigenvector of A?

   A. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

   B. $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

   C. $\begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$

   D. $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

   E. None of the above

10. The variance of a constant random variable is:

    A. 0

    B. 1

    C. Depends on the constant

    D. Infinite

    E. None of the above

11. A sample mean is an unbiased estimator of:

    A. variance

    B. population mean

    C. standard deviation

    D. median

    E. None of the above

12. Suppose $P(A) = 0.3$, $P(B|A) = 0.8$, $P(B) = 0.5$. Find $P(A|B)$.

    A. 0.3

    B. 0.48

    C. 0.5

    D. 0.76

    E. None of the above

13. Two events A and B are disjoint with
$P(A) = 0.3, P(B) = 0.5$.
What is $P(A \cap B)$?
   A. 0
   B. 0.15
   C. 0.8
   D. 0.2
   E. No correct answer

14. Let X be a random variable, uniformly distributed on the interval $[2, 6]$.
What is $P(X > 5)$?
   A. 0.2
   B. 0.25
   C. 0.5
   D. 0.75
   E. None of the above

15. In a medical test, a disease affects 1% of the population.
The test has sensitivity 95% and specificity 90%.
If a patient tests positive, what is the approximate probability they actually have the disease?
   A. About 75%
   B. About 25%
   C. About 9%
   D. About 1%
   E. None of the above

16. A bag contains 8 red, 4 blue, and 3 green balls.
Two balls are drawn without replacement.
What is the probability that both are blue?

A. 1/26
B. 2/91
C. 2/35
D. 1/13
E. None of the above

17. Let $f(x) = cx$ for $0 \leq x \leq 2$,
and $f(x) = 0$ otherwise.
For which value of c is $f(x)$ a
valid probability density function?

A. 1/2
B. 1
C. 1/4
D. 2
E. None of the above

18. A Binomial random variable X
has mean 4 and variance 2.
What is $P(X = 3)$?

A. $\frac{9}{32}$
B. $\frac{5}{32}$
C. $\frac{11}{32}$
D. $\frac{7}{32}$
E. None of the above

19. Given a sample mean $\overline{x}$, a population mean $\mu_0 = 18$, a known population variance 16, and a sample size n from a normal population, we perform a two-tailed Z-test for $H_0 : \mu = 18$ against $H_1 : \mu \neq 18$ at a significance level of 0.10. The critical value is $z_{0.05} \approx 1.645$. Which of the following data will lead us to reject H_0?

A. $\overline{x} = 17$, $n = 64$

B. $\overline{x} = 18$, $n = 100$

C. $\overline{x} = 19$, $n = 25$

D. $\overline{x} = 20$, $n = 9$

E. None of the above

20. The number of emails a person receives per hour follows a Poisson distribution with mean 3. Given that they receive at least one email in an hour, what is the probability that they receive exactly two emails in that hour?

A. $\dfrac{3}{e^3 - 1}$

B. $\dfrac{9}{2(e^3 - 1)}$

C. $\dfrac{9}{e^3 - 1}$

D. $\dfrac{9}{2e^3}$

E. None of the above